Proceedings of MACAS – 2015

International Symposium of

Mathematics and its Connections to the Arts and Sciences

10th Anniversary of the MACAS- Symposia

28th – 30th May 2015, University of Education Schwäbisch Gmünd, Germany

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Franzbecker

April 2016

Published by Verlag Franzbecker
Hildesheim

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ISBN 978-3-88120-760-7

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www.franzbecker.de

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Introduction

The symposium series MACAS have been founded in 2005 by an international group of researchers at the University of Education Schwäbsich Gmünd, Germany. In 2007, the 2nd Symposium was hosted by the University of Southern Denmark. In 2009, the 3rd Symposium has moved to the North America, to Uniersité de Moncton, New Brunswick, Canada. In 2015, MACAS is celebrating its tenth anniversary returning to its roots in Schwäbsich Gmünd for this special occasion.

The MACAS-vision is based on the goal to achieve a humanistic way of education that is combining various disciplines in one curriculum – an approach, which has been suggested by Renaissance philosophers. According to this philosophical notion, the aim is to educate students, in the way to enable them to pursue diverse fields of study, while getting insight into the aesthetic and scientific connection between arts and science. In view of the challenges of the 21st century, a modern education with a focus on inter- and multi-disciplinarity has gained a new and larger importance. In this perspective, the field of mathematics is taking over a key role through its connections to all other disciplines and can serve as a bridge between them. This holistic interdisciplinary and transdisciplinary approach is the heart of the MACAS (Mathematics and its Connections to the Arts and Sciences) philosophy.

The MACAS-2015 was targeting in particular scientists from mathematics, science, arts, humanities, philosophy, educational sciences and other disciplines that are scientifically connected to mathematics. The main idea of the MACAS symposia was to bring at one table scientists who are interested in the connection between arts and science in educational curriculum, while emphasizing on, as well as researching about, the role of mathematics. This role can be considered from different viewpoints, as previous MACAS activities have shown. Thus, at MACAS-2015 these different approaches and viewpoints between mathematics, arts and science, we pooled together, so that possible synergies and paths for future collaborations could be discovered.

This implied the following focus areas:

- Theoretical investigation of the relation between mathematics, arts and science
- Curricular approach to integrate mathematics and science
- Importance of the mathematical modelling and the inter-disciplinarity for the learning and studying of mathematics
- Meaning of arts and humanities for the understanding of the connection between arts, humanities and mathematics in ordinary daily situations
- Intercultural dimension of studying mathematics.

The MACAS-2015 also succeeded to bring together researchers, including emerging scholars, whose interests are focused on or connected to these fields of study. The conference thus promoted sharing scientific expertise, initiated new cooperations and enabled the reflection on commonalities and differences between different viewpoints.

The 3-days scientific program (http://www.macas.ph-gmuend.de/?page_id=8) featured 5 keynote presentations by the scholars from Canada, Denmark, Germany, and Switzerland, as well as 11 oral communications by researchers from Canada, China, Denmark, Germany and Switzerland. The MACAS proceedings present 16 peer-reviewed papers grouped in six sections.

In the plenaries section, the paper by Uffe Thomas Jankvist (Denmark) discusses *teaching modules on aspects of the History, Application, and Philosophy of mathematics* which allowed upper-secondary mathematics students to work with historical primary sources. George Gadandis (Canada) shared his *work with elementary schools, collaborating with teachers to develop an aesthetic dimension for engaging young children with big math ideas, while looking at parallels between "story" and "mathematics".* By looking into connections between mathematics and physics, Jean-Luc Dorier (Switzerland) describes *students' learning about vectors and translation in mathematis and physics, while inverstigating if they make the connection between these concepts introduced in different disciplines.* Gesche Pospiech (Germany) analyses the interplay *mathematics - physics* while looking at *possible sources of the often complained deficiencies of students in applying mathematical elements in physics.*

In the first section presenting oral talks, connections between mathematics and science were investigated by Simon Zell (Germany) who studied the *use a motion sensor to explore common aspects in mathematics and physics as possible way to enhance learning in upper secondary level.* The paper by Claus Michelsen (Denmark) *focused on how to strengthen the educational relations between mathematics and biology by interdisciplinary teaching centred on modeling activities.*

The second section deals with connections between mathematics and language. Namely, Shuzhu Gao et al. (China) discussed *vocabularies or terms in Chinese Mathematical Curriculum referring to the characters, words and phrases that indicate the mathematical objects and could be the sources of obstacles relevant to the understanding of mathematical concepts.* Silke Ladel and Julia Knopf (Germany) *showed basic principles for interdisciplinary lessons of Mathematics and language using the example of text message-communication.*

The third section is devoted to papers exploring connections between mathematics and arts. Xavier Robichaud and Viktor Freiman (Canada) analyze connections between music and mathematics, while discussing *some of the possible gains that could be obtained in making more explicit connections between the two disciplines in our classrooms by using technology*. Hans Peter Nutzinger (Germany) challenges views *that music is a subject, which you can learn and perform with delight, whereas mathematics often provokes a rather anxious feeling in learners as well as in many teachers* and suggests some possible ways in changing these beliefs. Dietmar Guderian (Germany) presents *modern examples of some mathematics applications in art like: mirroring, patterns and numbers, combinatorial analysis, hazard, parallels in mathematics/science and art, computer science, "prescientific mathematics" and concrete art, aesthetics of information, falsification.*

The fourth section looks into connections between mathematics and technology. The paper by Reinhard Oldenburg (Germany) analyses *functions a a key concept both of mathematics and of computer science while introducing several contexts in which the power of functions in computer science becomes more visible.* Viktor Freiman et al. (Canada) reviewed several initiatives implemented in New Brunswick, Canada by connecting technology and other disciplines by using robotics, online problem-soving, as well as Wiki collaborative environements. *The attention needs to be shifted to the development of transdisciplinary skills, often referred as 21st century skills, while looking specifically at the context of mathematics education and digital competences.*

The fifth section concludes the proceedings by investigating new paths in pedagogy. Dominic Manuel et al. (Canada) describe *regional differences in mathematics teaching and underlying* *pedagogies in Canada, and relate these differences to student achievement in mathematics.* Hans Walser (Switzerland) brings *different aspects of equivalence by dissection: variations on the theorem of Pythagoras, differences between methods and creativity, symmetry, optimizing, rational and irrational rectangles, color and esthetics.* Elena Klimova and Sabine Prinz, from Germany, analyse mathematical excursion as a 'window' to look for *various opportunities given by nature just outside the window.* Their paper *(1) illustrates a concept for the promotion of mathematically gifted and interested pupils in reference to popular theories about this issue and (2) investigates the positive effects of a mathematical excursion on the development of mathematically gifted and interested pupils.*

The overall success of the Symposium as result of a very productive scientific work magnificently supported by the great enthousiasm, devotion and hospitality of the local organising team lead by Professor, Dr. Astrid Beckmann, President of the University promots for continuation of the MACAS symposia in the coming years. The 5th one is planned in 2017 in Copenhagen, Denmark.

Astrid Beckmann, Claus Michelsen and Viktor Freiman, co-chairs of the symposium

Plenaries

PRIMARY HISTORICAL SOURCES IN THE TEACHING AND LEARNING OF MATHEMATICS – SHORT AND LONG TERM EFFECTS

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Abstract: *The study of primary historical sources is often described as a rewarding pursuit worth the effort, despite being extremely demanding for both teachers and students. In the present talk, focus shall be on recent empirical research findings from a Danish study and on the short and long term effects for students of having been exposed to readings of historical primary sources. The Danish study revolved around two specially designed, so-called, HAPh-modules, which are teaching modules on aspects of the History, Application, and Philosophy of mathematics. One of these modules concerned the early history of graph theory and its later application to shortest path algorithms, and the other concerned the history of Boolean algebra and its later application to electric circuit design. Upper secondary mathematics students, exposed to readings of primary source material as part of these modules in 2010-11, illustrate the short term effects; while undergraduate mathematics students exposed to the same material in 2012, and interviewed in 2015, illustrate the long term effects.*

Introduction

> Among the various possible activities by which historical aspects might be integrated into the teaching of mathematics, the study of an original source is the most demanding and the most time consuming. In many cases a source requires a detailed and deep understanding of the time when it was written and of the general context of ideas; language becomes important in ways which are completely new compared with usual practices of mathematics teaching. Thus, reading a source is an especially ambitious enterprise, but [...] rewarding and substantially deepening the mathematical understanding. (Jahnke et al., 2000, p. 291)

The above quote is taken from the ICMI-Study on *History in Mathematics Education* from 15 years ago, but is still as true today as back then. What Jahnke and colleagues point out is that although reading and working with primary historical – or original - sources in mathematics education is not an easy and straightforward task to undertake, it is certainly one which is worth the effort. As is also implicitly addressed in the quote is that the inclusion of original sources may serve more than just one educational purpose. A now somewhat common distinction is that of history of mathematics, including the studying of original sources, serving mainly as a 'tool' for the teaching and learning of mathematics, or as a 'goal' (e.g. Jankvist, 2009; 2014a).

History as a *tool* concerns the learning of so-called mathematical *in-issues*, e.g. that the study of original sources can teach students mathematical concepts, mathematical ideas and notions, theorems, proofs, etc. and that history in this respect can deepen their "mathematical understanding" (cf. quote above). Other arguments include that original sources offer a truer mode of presentation when compared to that of textbooks, which usually goes: definitions, theorems, proofs, (constructed) examples of application, while the actual historical development often is the reverse – both on the scale of details, but also on a scale of mathematical topics (e.g. Jankvist, 2014). Furthermore, with original sources the introduction of abstract notions is motivated mathematically, and may therefore be easier for students to comprehend (Barnett et al., 2014).

On the other hand, if it is considered a *goal* in itself to develop students' images of mathematics as a (scientific) discipline and their mathematical 'awareness', then original sources have equally much to offer in terms of addressing the metaperspective issues – or *meta-issues* – of mathematics. For example, original sources can illustrate that mathematics actually comes from somewhere, and is brought to life by human beings, that it interacts with culture, society, and other disciplines (and a bunch of other meta-issues, e.g. see Jankvist, 2009). Of course, to develop 'awareness' of such matters, a given original source must be placed in its historical setting, etc., as also pointed to in the quote from Jahnke and colleagues. Finally, from a meta-issue as well as an educational point of view, Fried (2001) points out that an original source is not 'pre-digested', meaning that it is open to students' own interpretations, which a secondary source often is not.

Of course, the above distinction between history as a tool and history as a goal should not be understood as this being an either/or. As we shall see, the main purpose can certainly be one of history as a goal where the meta-issues aimed for cannot be reached without learning about some mathematical in-issues along the way. (And the other way around is certainly also possible.)

Design principles of the HAPh-modules

The modules to be described are placed in a Danish setting of being concerned with students' development of mathematical competencies and so-called 'overview and judgment' regarding the subject of mathematics (Niss & Højgaard, 2011). While mathematical competencies (e.g. mathematical thinking, reasoning, problem handling, etc.) comprise a "well-informed readiness to act appropriately in situations involving a certain type of mathematical challenge", the three types of overview and judgment are "'active insights' into the nature and role of mathematics in the world" and Niss and Højgaard state that "these insights enable the person mastering them to have a set of views allowing him or her *overview and judgement of the relations between mathematics and in conditions and chances in nature, society and culture*" (Niss & Højgaard, 2011, pp. 49, 73, italics in original). The three types of overview and judgment (OJ) are:

- OJ1: the actual application of mathematics in other subject and practice areas – in particular the actual application of mathematics to extra-mathematical purposes within areas of everyday;
- OJ2: the historical development of mathematics, both internally and from a social point of view – in particular the fact that mathematics has developed in time and space, in culture and society;
- OJ3: the nature of mathematics as a subject – in particular that mathematics as a subject areas has its own characteristics and that some of these characteristics it has in common with other subject areas, but also that some of them are unique.

The HAPh-modules were based on an idea of having aspects of these three dimensions in one and same teaching module for upper secondary school. That is to say, to design a teaching module involving *H*istory of mathematics, *A*pplications of mathematics, and *Ph*ilosophy of