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RESEARCH CONSIDERATIONS FOR INTERDISCIPLINARY WORK ON MATHEMATICS AND ITS CONNECTIONS TO THE ARTS AND SCIENCES

Astrid Beckmann, Claus Michelsen, Bharath Sriraman

Organizers of the Symposium

INTRODUCTION

The First International Symposium of Mathematics and its Connections to the Arts and Sciences (MACAS1) was held at the University of Education - Schwäbisch Gmünd, Germany from May 19-21st, 2005. The Symposium was the outcome of the continued collaboration between some participants of Topics Study Group 21 at the 10th International Congress of Mathematics Education (ICME-10), held in Copenhagen in July 2004. The organizers of this Symposium believe the utopian goal is to create a humanistic approach of education, one that unifies various strands of the curricula as opposed to dividing it. Ideally the purpose of education is to create well-rounded individuals akin to the great thinkers of the Renaissance. That is, individuals who are able to pursue multiple fields of research and appreciate both the aesthetic and structural/ scientific connections between the arts and the sciences. In this endeavor mathematics can be thought as a unifying bridge, whose underlying structure intertwines multiple strands in the mosaic of Knowledge.

The goals of the Symposium were (1) to bring together researchers who share this vision of unifying the arts, mathematics, and the sciences in the school and university curricula; (2) to articulate a common research agenda; (3) to create networks of researchers with common interest; (4) to focus on creating a scholarly publication through which the work of the group may be transmitted.

The question then is: were these goals accomplished? What were the outcomes of this Symposium? And how can we proceed with research in this area of knowledge development? What are some of the practical considerations we need to take into account for implementation of research based findings into the classroom. We reflect on the organizing, proceedings and outcomes of the Symposium in this editorial. The organization of the Symposium began in October of 2004. Initially, it seemed almost impossible to organize this in such a short space of time, but in hindsight it seems that the time pressure was almost a necessary condition to accomplish our goals. The Symposium was advertised worldwide through e-mail networks, mailing flyers, word of mouth, numerous list serves, and mathematics/mathematics education web-sites of universities and professional organizations. The advertisements resulted in receiving numerous papers from different regions of the world. These proceedings will

reflect both the scholarly and geographical diversity of the Symposium. We thank all the participants for making the journey to Schwäbisch Gmünd and contributing to the richness of the scholarly experience.

AN OVERVIEW AND DISCUSSION OF CONTRIBUTED PAPERS

All papers appearing in these proceedings[1] were refereed by the Editors and by external reviewers in some cases, where the contents of the manuscript were beyond the realm of our expertise. The proceedings have been arranged thematically according to the presentations that occurred during the Symposium. There were three plenary lectures, arranged in reverse order to provide variety.

Richard Lesh and Bharath Sriraman's paper is based on concluding plenary talk given by Richard Lesh on making science practical versus making practice scientific. In this paper Lesh & Sriraman put forth the models and modeling perspective (MMP) of learning based on the ongoing work carried out by Lesh & Doerr (2003). Lesh & Sriraman essentially propose a paradigmatic shift in math and science education, one that takes into account that complex systems characterize the student's world today, which implies that mathematics and science education should focus on both introducing such complex systems to students via model eliciting activities (MEA's) and then studying the conceptual systems that student's develop within a community of learners in their attempts to resolve such problematic and complex situations. The authors suggest that unlike good theories which don't change much, models and modeling perspectives are adaptable, driven by reality and practice, changeable and do not fit neatly into any single theory analogous to how reality functions.

Bharath Sriraman's plenary paper focuses on philosophy as a bridge between mathematics, the arts and the sciences. The paper traces the historical lineage of connections between mathematics, arts and science starting with the Socratics onto the Renaissance onto the 20th century. This paper focuses on the major shifts in perspectives that occurred in the arts and sciences over the course of history due to the influence of philosophy and theology during specific time periods. Sriraman demonstrates that these shifts often resulted in new paradigms for science and the creation of new mathematics. The symbiotic relationship between mathematics, arts and sciences with philosophy as the underlying scaffolding is explored in this paper. The paper also investigates the notion of *polymathy* and outlines its basic principles with implications for education today.

At the end of the proceedings, Dietmar Guderian's plenary paper focuses on mathematics in contemporary arts and culture. Guderian argues that although numerous examples of the integration of mathematics with other disciplines are found in the classical arts, most of these convey a clichéd view of mathematics' connections with the arts and culture. Guderian presents numerous approaches

in the arts which often unconsciously or unintentionally visualize mathematical thinking and express thoughts, which are difficult to comprehend (the limits of parallelism according to Euclid, infinity in the finite, limitation without boundary, etc.). Contemporary examples of artists who visualized mathematical thoughts at the same time as mathematicians or even before (chaos, ambiguous solvability, etc) are used to show the connections of arts with mathematics even though art is not required to show the correctness of a mathematical statement or an idea.

The panel papers begin with connections of Mathematics with the Arts. Mohammed Nou (Egypt) outlines the concept of symmetry as a critical schema pervasive within ancient civilizations. Several examples of symmetric artifacts from Ancient Egypt are presented in the paper. The next paper Geomart by Rachel Filo, Merav Yarkoni and Jenny Grayfer (Israel) is based on van Hiele's theory of learning development in geometry and suggests that art can serve as a powerful teaching and learning tool in an interdisciplinary curriculum. Heinz Schumann (Germany) introduces us to the prototype of Cabri 3D as a cutting edge tool for modeling three dimensional objects. The paper illustrates the exciting possibilities of using this tool for connections between the visual architectural world and three-dimensional geometry. These papers suggest that the historic (e.g., the monuments of ancient Egypt) and the contemporary (e.g., the rebuilding of monuments) can be intertwined and mathematically explored with new virtual technologies and create exciting possibilities for the teaching and learning of mathematics.

The panel papers on Mathematics and Aesthetics consist of contributions from Veronica Preiss (Norway) and Akihiro Matsuura (Japan). Preiss discusses her ongoing exploratory work with fifth grade children. By designing open ended projects for the children, which involve visual, auditory and kinesthetic manipulations, Preiss finds that children are both creative and adept with generating descriptions of a circle, quite different from the theoretical definition found in books. This paper highlights that inductive methods of discovery learning are time consuming but ultimately rewarding. Matsuura, a perfoming artist and a mathematician delves into the mathematics of Dupin cyclides to create visual instruments that illustrate illusory motions. Matsuura reports on his experiences of creating physical objects based on the mathematics of cyclides to create mind boggling visual performances that show illusory phenomena. Both these papers illustrate the sensory dimension of mathematics possible at the elementary and university levels.

The panel papers on Mathematics and Literature and Language present numerous didactic and research possibilities for those interested in forging interdisciplinary connections between these two disciplines. Astrid Beckmann (Germany) presents historic and contemporary examples of mathematics

inherent in literary texts. This paper also introduces the software called MATEX which connects short literary texts and mathematical learning. Jyotsna Joshi (India) contributes an unusual paper on the statistical study of English words. Joshi claims that existing lexicographic English dictionaries are difficult for new learners of the language, and proposes a dictionary based on the "order" and "potential" of one letter, two letter, three letter and four letter words. The presentations in this panel show the complexity of the connections between mathematics and literature. Beckmann suggests that literature can motivate the study/discovery of mathematics as shown in the texts, but she also emphasizes that literary texts have to be chosen and studied carefully. For instance there are numerous useful texts that can lead to different kinds of mathematical models and structures. In some texts the mathematics background can lead to a deeper literary understanding (e.g. The rider on the white horse). Some texts can also be chosen in mathematics lessons to alleviate negative views of mathematics in the text. In all instances the researcher should be aware of the influence of such activities with the literary content. Joshi's paper begs the question of measuring or testing the effectiveness of these new dictionaries and handy workbooks based on "order" and "potential". Joshi's paper begs the question of measuring the effectiveness of these new dictionaries and handy workbooks based on "order" and "potential".

The papers on Mathematics and Philosophy investigate philosophical problems in our conceptions (and acceptance!) of mathematics as an immutable truth. Jessica Carter (Denmark) puts forth the philosophy of constructive realism in which objective existence to mathematical objects is only granted after mathematicians have conceptually constructed it. This paper touches on some ideas recently proposed by Penelope Maddy (1997) on naturalism in mathematics. Wolfgang Mückenheim (Germany) attacks Platonic notions of attributing existence to objects described in mathematical ideas which in fact are impossible to physically construct. Mückenheim suggests that the natural numbers are not independent of the physical constraints imposed by the laws of the universe. Steffen Iversen (Denmark) investigates authentic situations which allow for philosophical competencies to develop in the mathematics classroom, and presents a conceptual model for further developing interdisciplinary connections between mathematics and philosophy. Iversen's views are pragmatic in nature and warn us to be wary of implementing interdisciplinary reform prescribed by a governmental body (the Danish ministry of Education) without fully thinking of the didactical consequences, both positive and negative. These three papers open numerous questions. Is constructive realism a form of psychological Platonism? That is, if mathematical objects are granted existence after their have been constructed and are used by other mathematicians in a particular area of mathematics, then doesn't this ascribe a platonic reality towards these objects, albeit psychological and not physical? Mückenheim's paper in a sense attacks the mental comfort we garner from

existence theorems in mathematics with properties that are physically intangible. The readers are urged to think about and comment on the interesting ideas in this paper. Finally, Iversen's paper suggests that we analyze the long term effectiveness of introducing mathematics and mathematical thinking via philosophy and vice versa. This is a very classical approach which might be relevant to our times since students are often confronted with ethical dilemmas with the advent of new technologies. Thus being able to reason philosophically (and critically) is certainly a valuable trait to cultivate in the school curriculum.

In the panel on Mathematics and Didactical Issues, Miroslav Lovric (Canada) addresses the pros and cons of requiring writing assignments of university students in introductory mathematics courses. Although encouraging students to write about mathematics brings forward the picture of mathematics as narrative, the hidden danger is student writing that only touches superficial aspects of the mathematical ideas. Lovric argues that university teachers should be critical of student writing and encourage student's to pursue and convey the mathematical ideas in depth. Viktor Freiman and Nicole Lirette-Pitre (Canada) propose new ideas which make use of cutting-edge technology available in the online world to create a forum where trans-disciplinary learning occurs. The task of training teachers to implement communication, information and communication technology, critical thinking, personal and social development, study and work habits, and culture and heritage in the K-12 curriculum, as required by the New Brunswick's new learning initiative, led Freiman and Pitre to using WIKI's for educational purposes. An interesting outcome of Freiman and Pitre's paper is that after the Symposium we have set up a MACAS WIKI, so that the participants can network with each other and pursue research ideas for the next Symposium. The WIKI is available at http://sciencematheducation.xwiki.com

The proceedings of the Mathematics and Music panel include Manfred Euler's (Germany) abstract on how non-linear dynamical systems are used to model processes of the auditory systems. However these dynamical systems are in turn governed by small integers and their ratios which can be generated by musical instruments, thus related to our musical perception. Günter Graumann (Germany) summarizes the classical Pythagorean theory of music, and connects the analysis of tones to the basic operations of addition, multiplication, division and logarithms, with implications for the teaching of mathematics via music in the secondary curriculum.

The panel on Mathematics and Physics comprised of four papers. Ivan Drazic (Croatia) proposes a teaching method for teaching sophisticated theories by using Richard Skemp's (1986) theoretical framework and the mathematical context of theory of distributions. Tine Golez (Slovenia) illustrates the use of motion detection devices and compatible software to introduce Calculus intuitively to students in Physics lessons, making difficult notions from

Calculus accessible to students. Claus Michelsen (Denmark) generalizes Golez's ideas by recommending and illustrating the use of model eliciting activities in general to introduce concepts in mathematics with intuitive connections to physics. Finally Astrid Beckmann and Arthur Litz (Germany) provide relevant and thought provoking didactical ideas tested in the classroom, which illustrate how teachers can facilitate the learning of the concept of function through different experimental activities.

CONCLUDING POINTS

We believe that MACAS1 achieved three of the four goals we spelled out at the beginning of this editorial. We did accomplish the goal of bringing together researchers who share a vision of unifying the arts, mathematics, and the sciences in the school and university curricula. We now have a network of researchers with some common interests. The Symposium proceedings are the start to creating a scholarly publication through which the work of the group is transmitted. However we have not accomplished the goal of articulating a common research agenda. We think that the Symposium was best characterized by the word diversity, which shows the need for the development of a common theoretical framework under which researchers can actively pursue the myriad ideas. We also think that in the future, we would like to see more presentations based on empirical investigations which operationalize, model and study the rich ideas presented in Schwäbisch Gmünd. The next Symposium is scheduled for May 2007 in Odense, Denmark. We hope that the proceedings will initiate further collaboration, networking and result in joint research reports for the next Symposium.

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Endnotes

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